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Higher Order Diffraction Characteristics of Fiber Bragg Grating

Sunita P. Ugale, Vivekanand Mishra

Abstract — **The effect of grating saturation on higher order diffraction characteristic of FBG is investigated by using Coupled mode theory. Grating saturation effects were considered in the index distribution model showing the significant influence on the coupling process and hence on the reflectivity characteristics of FBG. Maximum reflectivity curves for first and higher order diffraction of FBG are plotted for different values of saturation coefficient. The effect of change in length and change in refractive index are studied. The behavior of grating for higher order of diffraction is totally different than first order of diffraction. In saturated gratings, the higher order diffraction can be utilized for multiparameter sensing.**

Keywords — **FBG, Diffraction, Reflectivity, Saturation Coefficient, Higher Order Diffraction.**

I. Introduction

In fiber grating a periodic perturbation of refractive index along the fiber length is formed. These perturbations scatter light. It selectively reflects a narrow range of wavelength. Each time the light hits a region of higher refractive index, a bit is scattered backwards. If the wavelength matches the spacing of the high-index zone in the fiber, the waves scattered from each high index zone interfere constructively, producing strong reflections[1-3]. The wavelength selected is twice the distance between the lines written into the fiber because the light wave has to go through the region between them twice. If Λ is the grating period, n_{eff} is the effective refractive index of grating in the fiber core as shown in Fig.1, then the reflected wavelength is called Bragg Wavelength and is defined by the relationship,

$\lambda_B = 2n_{eff}\Lambda$ (1)

Where $n_{eff} = \frac{n_2+n_3}{2}$

We assume that the grating is uniform along the direction. The index inside the core after the FBG has been printed can be expressed by

$n(z) = n_{co} + \Delta n_0 n_d(z)$ (2)

Where n_{co}- core refractive index,
$n_d(z)$-index variation function
Δn_0- maximum index variation

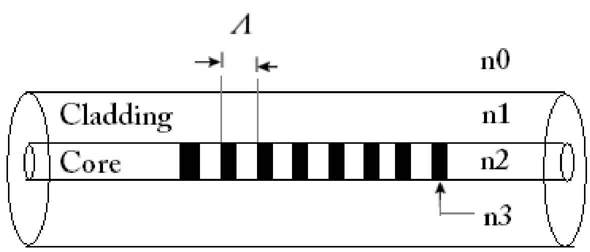

Fig.1. Fiber Bragg Grating Structure.

II. Modeling

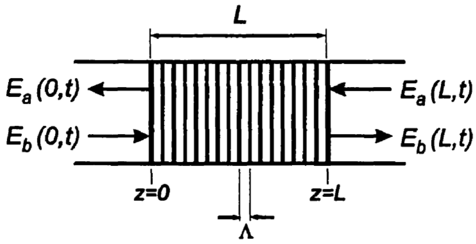

Fig.2. Propagating waves in Bragg grating

The fiber contains a Bragg grating, of length L and uniform pitch length Λ as shown in Fig. 2. The electric fields of the backward and forward propagating waves can then be expressed as[4-7].

$E_a(z, t)= A(z)\, e^{i(\omega t-\beta z)}$ (3)

$E_b(z, t)= B(z)\, e^{i(\omega t+\beta z)}$ (4)

For the backward and forward propagating waves, respectively

The coupled-mode equations describe their complex amplitudes, *A(z)* and *B(z)*

$$\frac{dA(Z)}{dz} = i\kappa B(z)e^{-2i(\Delta\beta)z} \qquad 0\leq z \leq L$$

$$\frac{dB(Z)}{dz} = -i\kappa^{*}A(z)e^{+2i(\Delta\beta)z} \qquad (5)$$

Where $\Delta\beta=\pi/\Lambda$, and Λ is the grating period and κ is the coupling coefficient. It is constant for uniform gratings and related to index modulation depth.

If we assume that both forward and backward waves enter the grating, then assume the boundary conditions *B(0) = Bo* and $A(L) = A_L$. Substituting these boundary conditions into equation 5, we can solve for the closed-form solutions and thus the z-dependence of the two waves.

$$a(z)=A(z)e^{-i\beta z}$$
$$b(z)=B(z)e^{i\beta z} \qquad (6)$$

The reflected wave, *a(0)*, and the transmitted wave, b(L) can be expressed by means of the scattering matrix

$$\begin{bmatrix} a(0) \\ b(L) \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a(L) \\ b(0) \end{bmatrix} \qquad (7)$$

Substituting *a(L)*and b(0) from equation 6 into equation 7 we get

$$S_{11} = S_{22} = \frac{iSe^{-i\beta 0L}}{-\Delta\beta sinh(SL) + iScosh(SL)}$$

$$S_{12} = \frac{\kappa}{\kappa^{*}}S_{21}e^{2i\beta 0L} = \frac{\kappa sinh(SL)}{-\Delta\beta sinh(SL)+ iScosh(SL)} \qquad (8)$$

Based on equations 7 and 8, the scattering matrix, we can obtain the transfer-matrix, or T-matrix equation.

$$\begin{bmatrix} a(0) \\ b(0) \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} a(L) \\ b(L) \end{bmatrix} \qquad (9)$$

Where

$$T_{11} = T^{*}_{22} = \frac{\Delta\beta sinh(SL) + iScosh(SL)}{iS} e^{-i\beta 0L}$$

$$T_{12} = T^{*}_{21} = \frac{\kappa sinh(SL)}{iS} e^{-i\beta 0L} \quad (10)$$

This matrix approach is effective at treating a single grating as a series of separate gratings each having reduced overall lengths and different pitch lengths, and describing each with its own T-matrix. Combining all the matrices yields the properties of the initial non-uniform grating [9,10]. The resultant system of matrices is treated as an individual matrix

$$[T_L] = [T_1][T_2] \dots [T_M] \quad (11)$$

Light passing through successive optical elements can be calculated by series of matrices, as such

$$\begin{bmatrix} a(0) \\ b(0) \end{bmatrix} = [T_M][T_{M-1}] \dots [T_1] \begin{bmatrix} a(L) \\ b(L) \end{bmatrix} \quad (12)$$

The characteristics response from Bragg Grating can be fully described by

1. The center wavelength of Grating λ_B
2. Peak reflectivity R_{max} of grating which occur at λ_B
3. Physical length of Grating L
4. Refractive index of core of optical fiber n_{co}
5. Amplitude of induced core index perturbation Δn

For a grating with uniform index modulation and period the reflectivity is given by

$$R(L,\lambda) = \frac{\kappa^2 \sinh^2(SL)}{\Delta\beta^2 \sinh^2(SL) + \kappa^2 \cosh^2(SL)} \quad (13)$$

Where R: Grating reflectivity as a function of both grating length and wavelength
L: total length of grating
κ: coupling constant, given by $\kappa=\pi\Delta n/\lambda$
Δβ: wave vector detuning, given by

$$\Delta\beta=\beta-(\pi/\Lambda)$$

β: fiber core propagation constant, given by $\beta=2\pi n_0/\lambda$

$$S=\sqrt{\kappa^2-\Delta\beta^2}$$

For light at the Bragg grating center wavelength, λ_B, there is no wave vector detuning and so $\Delta\beta = 0$ and $S=\kappa$
The reflectivity function then becomes [8, 10]

$$R(L, \lambda)=\tanh^2(\kappa L) \quad (14)$$

The grating saturation effect has significant influence on the coupling process. The FBG model, taking saturation into account, assumes that the power of UV laser source and the printing exposure time are unlimited. The index variation caused by defect centers bond breaking through UV light absorption can only be slightly modified because the **doping concentration is always limited.** This means the refractive index cannot always respond linearly to the printing conditions such as UV source power or exposure time. Saturation will not occur if the exposure is not great enough. Saturation will occur with high exposures, however, depending upon the doping concentration level. The **doping concentration limits the index variation to the maximum value Δn_0.** Here we define *d* as the *depth of saturation* (the width of the flat top in Fig. 3 caused by the saturation), Λ as the grating index perturbation period, and define *l=d/Λ* as the ***saturation coefficient***. We assume that the grating is uniform along the direction. The index inside the core after the FBG has been printed can be expressed by

$$n(z) = n_{co} + \Delta n_0 n_d(z) \quad (15)$$

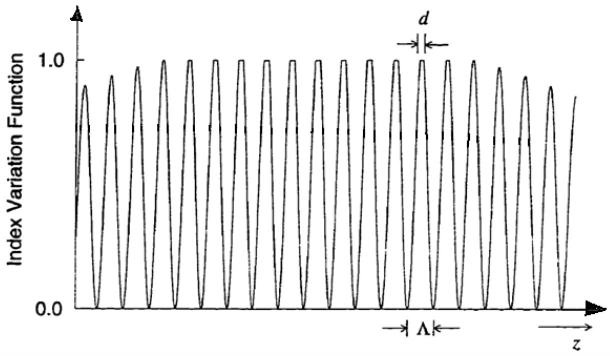

Fig.3. FBG Index variation function

Where n_{co} is the refractive index inside the fiber core, Δn_0 (which is positive) is the maximum index variation and $n_d(z)$ can be called the index variation function. It is given by [1]

$$n_d(z) = \begin{cases} \frac{1+\cos\left(2\frac{\pi}{\Lambda}z\right)}{1+\cos(\pi l)} & \left[-\frac{\Lambda}{2},-\frac{d}{2}\right]\cup\left[\frac{d}{2},\frac{\Lambda}{2}\right] \\ 1 & \left[-\frac{d}{2},\frac{d}{2}\right] \end{cases} \quad (16)$$

This can be expanded into Fourier series

$$n_d(z) = G_0 + \sum_{n=1}^{\infty} G_n \cos\left(n\frac{2\pi}{\Lambda}z\right) \quad (17)$$

Where

$$G_0 = l + \frac{1}{1+\cos(\pi l)}\left|1 - l - \frac{\sin(\pi l)}{\pi}\right| \quad (18)$$

And

$$G_n = \frac{2}{n\pi}\sin(n\pi l) + \frac{1}{1+\cos(\pi l)} \cdot \left\{\frac{\sin\pi(n-1)-\sin\pi(n-1)l}{(n-1)\pi} + \frac{2[\sin(n\pi)-\sin(n\pi l)]}{n\pi} + \frac{\sin\pi(n+1)-\sin\pi(n+1)l}{(n+1)\pi}\right\} \quad (19)$$

The maximum reflectivity of the grating is given as

$$R(L, \lambda)=\tanh^2(\kappa L) \quad (20)$$

For the first-order of diffraction, $\Delta n=\Delta n_0 G_1$
For the second-order of diffraction $\Delta n=\Delta n_0 G_2$
For the first order diffraction Bragg wavelength λ_B is

$$\lambda_B = 2n_{eff}\Lambda$$

For the second order diffraction,

$$\lambda_B = n_{eff}\Lambda$$

Where Λ is the grating period, and

$$n_{eff} \approx n_{co} + \Delta n_0 G_0$$

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III. SIMULATION RESULTS AND DISCUSSION

The zeroth-order coefficient G_0 along with the first, second, third, fourth and fifth order coefficients are plotted in Fig. 4 to illustrate how saturation causes energy redistribution among the different frequency components [1]. **G_0 controls the self-coupling coefficient and determines the mode coupling peak position.** It also determines which bounded-cladding mode is involved in the coupling process and explains grating movement after grating saturation. **The G_n values (n≥ 1) determine the cross-coupling coefficients**, so they have a great deal of influence on the grating coherence peak reflectivity (or peak transmission loss) point and the bandwidth. The higher order coefficients G_n involve coupling processes which form multiple main-peaks in the output spectrum.

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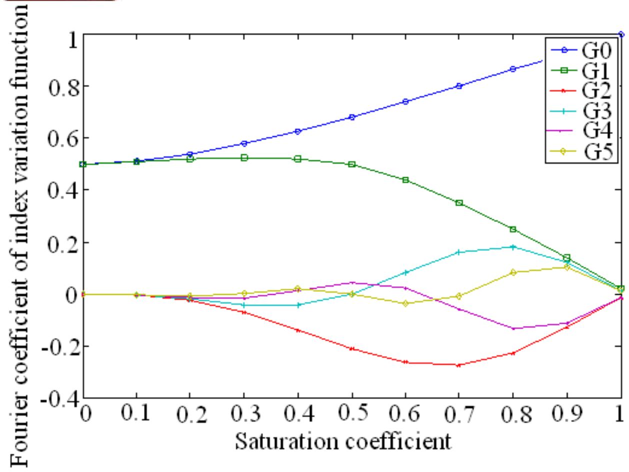

Fig.4. First six Fourier-series coefficients for the index variation function

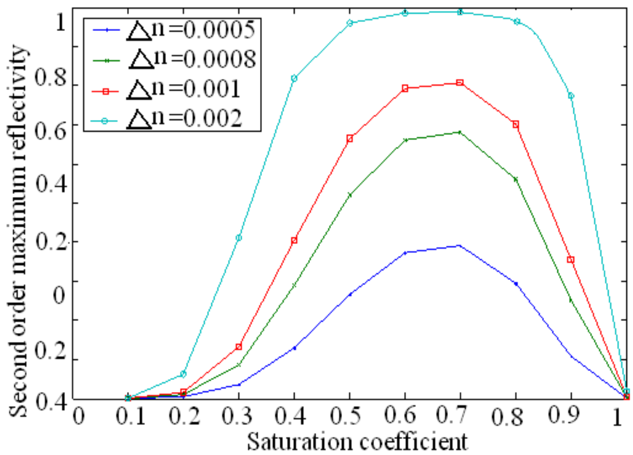

Fig.7. Second order Maximum Reflectivity Versus Saturation Coefficient for different modulation depth (Δn)

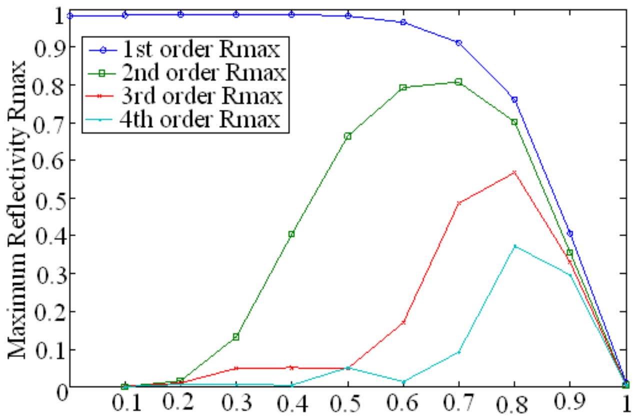

Fig.5. First, second, third, fourth order Maximum Reflectivity Versus Saturation coefficient

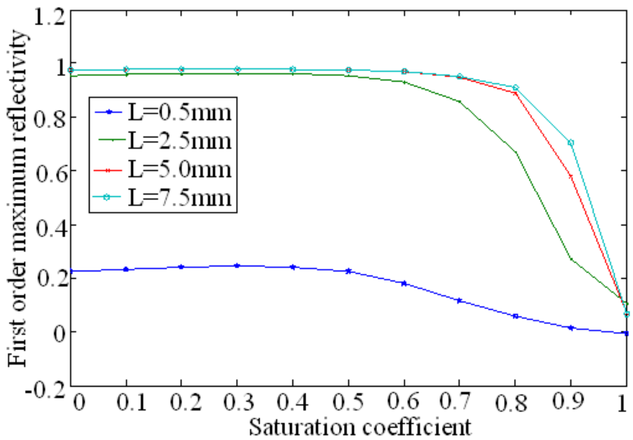

Fig.8. First order Maximum Reflectivity Versus Saturation Coefficient for different Length of Grating

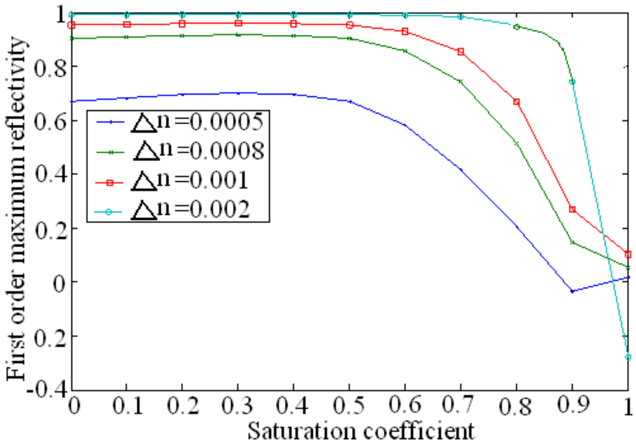

Fig.6. First order Maximum Reflectivity Versus Saturation Coefficient for different modulation depth (Δn)

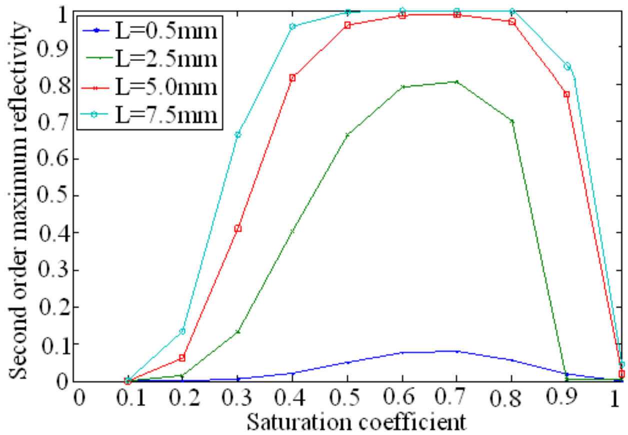

Fig.9. Second order Maximum Reflectivity Versus Saturation Coefficient for different Length of Grating

Fig. 5 shows the simulation results for the first, second, third and fourth order diffraction of FBG. The FBG parameters that are used are: Δn_0=0.001, Λ=0.5μm, L= 2.5mm and n_{co}=1.46 and the saturation coefficient is varied from 0 to 1 in step of 0.1.

There is no diffraction of second and higher degree in FBG before saturation. Second degree diffraction occurs almost after saturation coefficient of 0.2, third degree diffraction is significant after saturation coefficient of 0.5 and third degree diffraction is significant after saturation coefficient of 0.7, and this value increases as the order of diffraction increases.

Fig. 6 shows the simulation results for first order diffraction of FBG, and it is observed that when modulation depth Δn is increased, maximum reflectivity R also increased. Λ=0.512µm, L= 2.5mm and n_{co}=1.46 and Δn_0=0.0005, Δn_0=0.0008, Δn_0=0.001 and Δn_0=0.002

Fig. 7 shows the simulation results for first order diffraction of FBG, and it is observed that when Length of Grating L is increased, maximum reflectivity R also increased. Λ=0.512µm, L= 2.5mm and n_{co}=1.46 and Δn_0=0.001, L=0.5mm, L=2.5mm, L=5.0mm and L=7.5mm.

First and second order maximum reflectivity increases with increase in length and is almost constant after 5.0mm length.

Fig. 8 shows the simulation results for second order diffraction of FBG, and it is observed that when modulation depth Δn is increased, maximum reflectivity R also increased. Λ=0.512µm, L= 2.5mm and n_{co}=1.46 and Δn_0=0.0005, Δn_0=0.0008, Δn_0=0.001 and Δn_0=0.002

Fig. 9 shows the simulation results for second order diffraction of FBG, and it is observed that when Length of Grating L is increased, maximum reflectivity R also increased. Λ=0.512µm, L= 2.5mm and n_{co}=1.46 and Δn_0=0.001, L=0.5mm, L=2.5mm, L=5.0mm and L=7.5mm.

First and second order maximum reflectivity also increases with increase in modulation depth and becomes 100% after modulation depth of 0.002.

IV. Conclusion

We have modeled and studied Fourier-series coefficients for the index variation function of optical fiber grating. First to fourth order diffraction spectra of FBG were obtained by varying the length of grating and modulation depth.

Without saturation it was observed that reflectivity increases with increase in grating length as well as index difference. From the first to fourth orders Maximum Reflectivity versus Saturation coefficient graph, we can say that there is no diffraction of second and higher degree in FBG before saturation.

First and second order maximum reflectivity increases with increase in length and is almost constant after 5.0mm length. It also increases with increase in modulation depth and becomes 100% after modulation depth of 0.002.

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